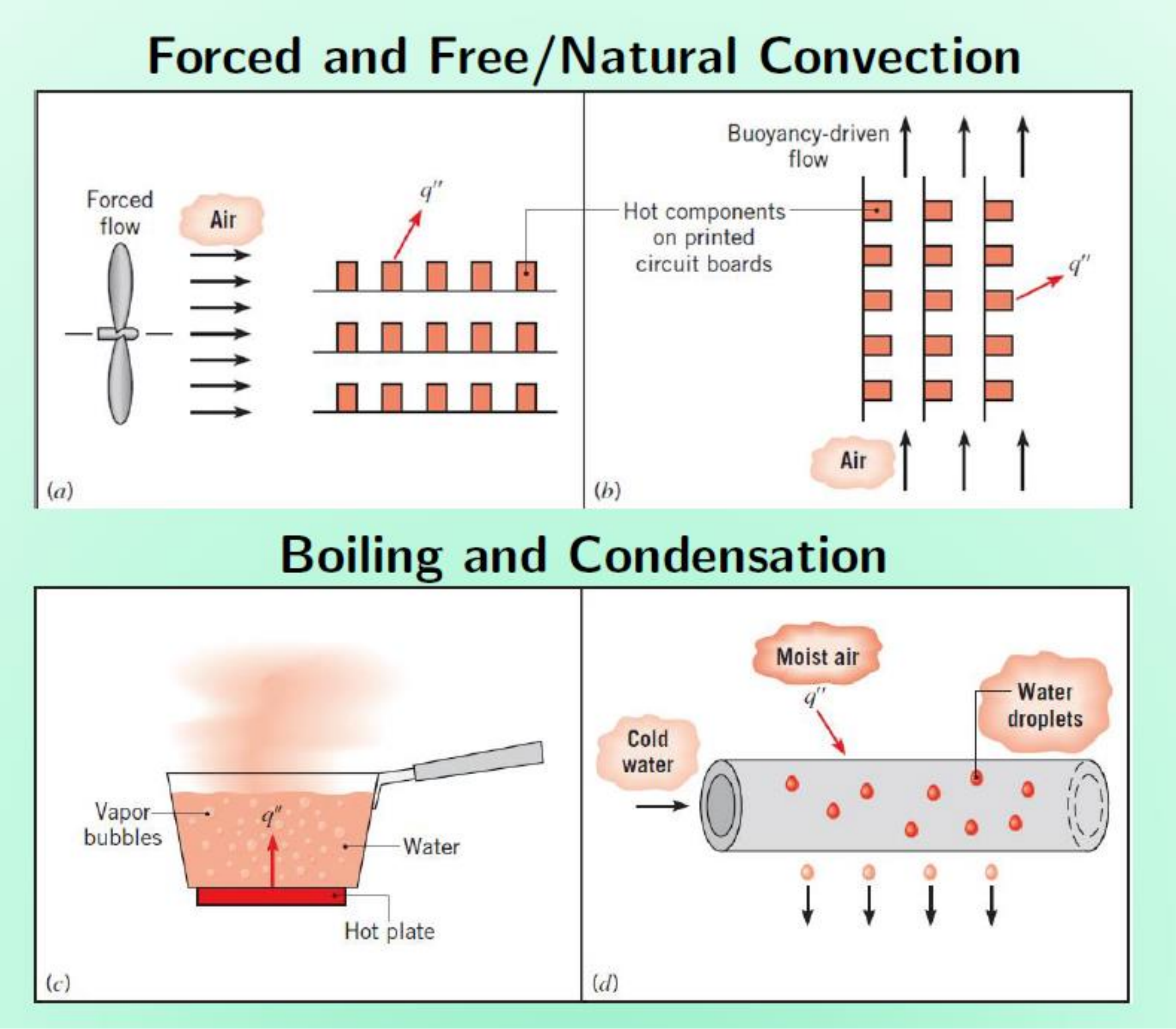
Heat Transfer

Unit III

Convection Heat Transfer

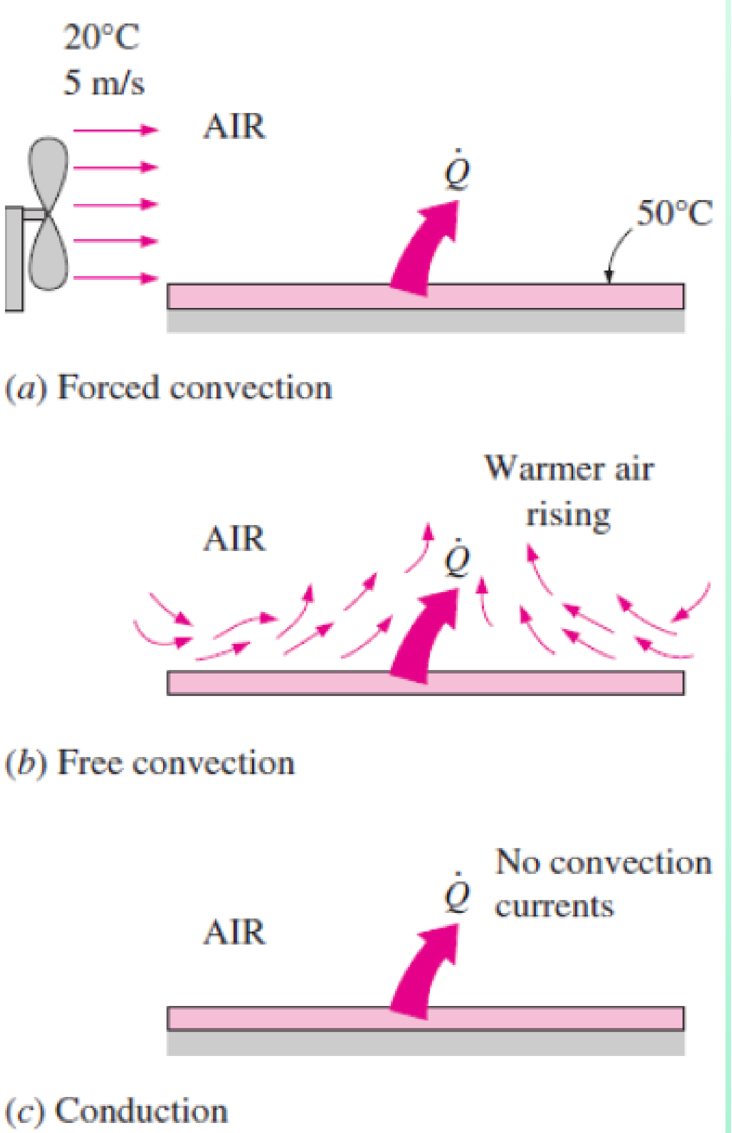

Forced and Free/Natural Convection
Forced flow
Air
q''
Hot components on printed circuit boards
Buoyancy-driven flow
q''
Air
(a)
(b)
Boiling and Condensation
Vapor bubbles
q''
Water
Hot plate
Moist air
q''
Water droplets
Cold water
(c)
(d)

Convective heat transfer involves

- fluid motion
- heat conduction

The fluid motion enhances the heat transfer, since it brings hotter and cooler chunks of fluid into contact, initiating higher rates of conduction at a greater number of sites in fluid. Therefore, the rate of heat transfer through a fluid is much higher by convection than it is by conduction.

Higher the fluid velocity, the higher the rate of heat transfer.



Convection heat transfer strongly depends on

- fluid properties: μ, k, ρ, C_p
- fluid velocity: V
- geometry and the roughness of the solid surface
- type of fluid flow (laminar or turbulent)

Newton's law of cooling

$$q_{conv} = hA_s\left(T_s - T_\infty\right)$$

T_∞ is the temp. of the fluid sufficiently far from the surface

Local heat flux

$$q''_{conv} = h_l\left(T_s - T_\infty\right)$$

h_l is the local convection coefficient

Flow conditions vary on the surface: q'', h vary along the surface.

The total heat transfer rate q:

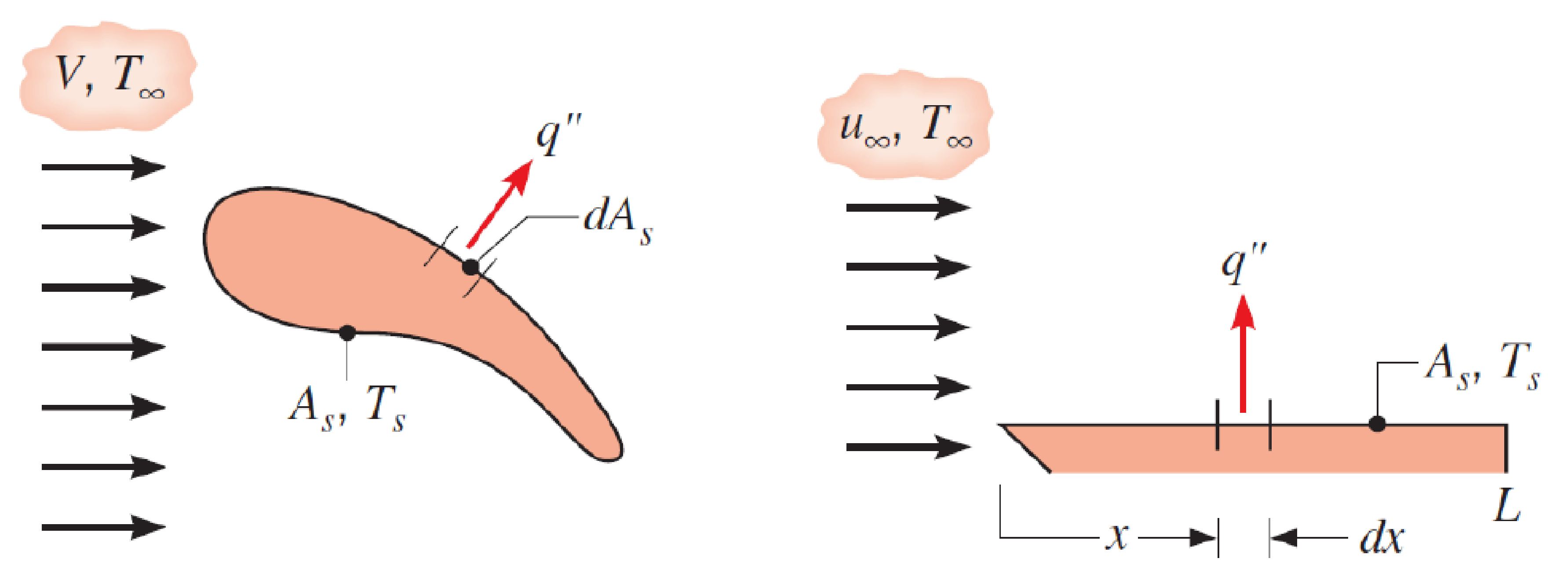
$$q_{conv} = \int_{A_s} q'' dA_s$$

$$= \left(T_s - T_\infty\right)\int_{A_s} h dA_s$$

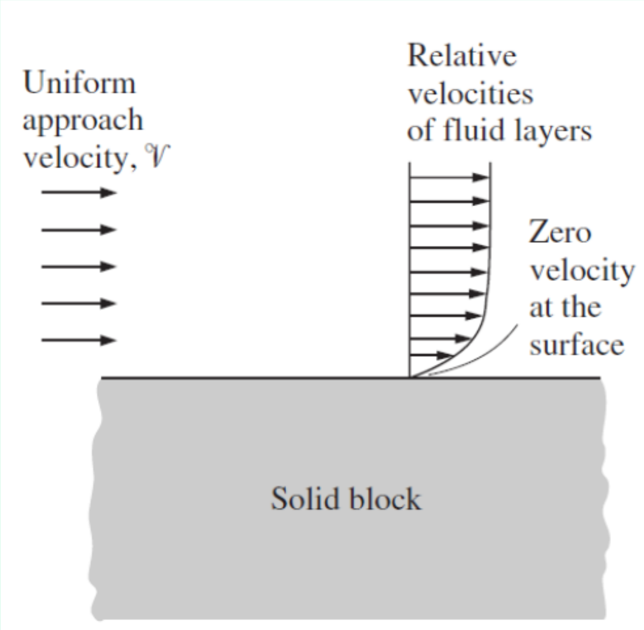
Defining an *average convection coefficient* $\bar{h}$ for the entire surface,

$$q_{conv} = \bar{h} A_s (T_s - T_\infty)$$

$$\bar{h} = \frac{1}{A_s} \int_{A_s} h dA_s$$



A fluid flowing over a stationary surface - no-slip condition



A fluid and a solid surface will have the same T at the point of contact, known as no-temperature-jump condition.

With no-slip and the no-temperature-jump conditions: the heat transfer from the solid surface to the fluid layer adjacent to the surface is by pure conduction.

$$q''_{conv} = q''_{cond} = -k_{fluid} \left. \frac{\partial T}{\partial y} \right|_{y=0}$$

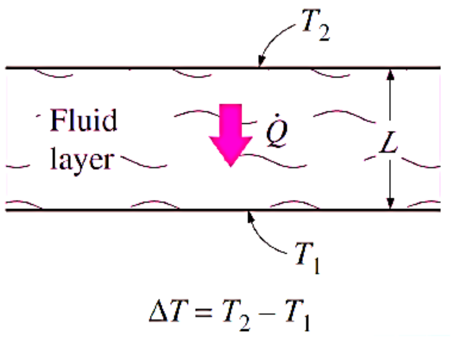
T represents the temperature distribution in the fluid $(\partial T/\partial y)_{y=0}$ *i.e.*, the temp. gradient at the surface.

$$q''_{conv} = h(T_s - T_\infty)$$

$$h = \frac{-k_{fluid} \left(\frac{\partial T}{\partial y} \right)_{y=0}}{T_s - T_\infty}$$

Nusselt Number

$$\text{Nu} = \frac{hL_c}{k_{fluid}}$$



Heat transfer through the fluid layer will be by convection when the fluid involves some motion and by conduction when the fluid layer is motionless.

$$q_{conv} = h\Delta T \qquad q_{cond} = k\frac{\Delta T}{L}$$

$$\frac{q_{conv}}{q_{cond}} = \frac{h\Delta T}{k\Delta T/L} = \frac{hL}{k} = \text{Nu}$$

$$\text{Nu} = \frac{q_{conv}}{q_{cond}}$$

Nusselt number: enhancement of heat transfer through a fluid layer as a result of convection relative to conduction across the same fluid layer.

$\text{Nu} >> 1$ for a fluid layer - the more effective the convection

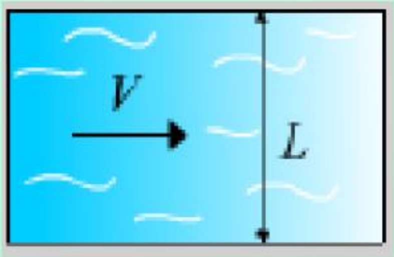
$\text{Nu} = 1$ for a fluid layer - heat transfer across the layer is by pure conduction

$\text{Nu} < 1$???

Reynolds Number

Osborne Reynolds in 1880's, discovered that the flow regime depends mainly on the ratio of the inertia forces to viscous forces in the fluid.

Re can be viewed as the ratio of the inertia forces to the viscous forces acting on a fluid volume element.



$$Re = \frac{\text{Inertia forces}}{\text{Viscous}} = \frac{V L_c}{\nu} = \frac{\rho V L_c}{\mu}$$

Prandtl Number

- Shape of the temp. profile in the thermal boundary layer dictates the convection heat transfer between a solid surface and the fluid flowing over it.
- In flow over a heated (or cooled) surface, both velocity and thermal boundary layers will develop simultaneously.
- Noting that the fluid velocity will have a strong influence on the temp. profile, the development of the velocity boundary layer relative to the thermal boundary layer will have a strong effect on the convection heat transfer.

$$Pr = \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of heat}} = \frac{\nu}{\alpha} = \frac{\mu C_p}{k}$$

$$\frac{\delta}{\delta_t} \approx \text{Pr}^n$$

n is positive exponent

- Pr $\cong 1$ for gases $\Longrightarrow$ both momentum and heat dissipate through the fluid at about the same rate.
- Heat diffuses very quickly in liquid metals (Pr < 1).
- Heat diffuses very slowly in oils (Pr > 1) relative to momentum.
- Therefore, thermal boundary layer is much thicker for liquid metals and much thinner for oils relative to the velocity boundary layer.

$$\delta = \delta_t \text{ for Pr } = 1$$

$$\delta > \delta_t \text{ for Pr } > 1$$

$$\delta < \delta_t \text{ for Pr } < 1$$

$$\text{Pr} = \frac{\nu}{\alpha}$$

Typical ranges of Pr for common fluids

Fluid	**Pr**
Liquid metals	0.004-0.030
Gases	0.7-1.0
Water	1.7-13.7
Light organic fluids	5-50
Oils	50-100,000
Glycerin	2000-100,000

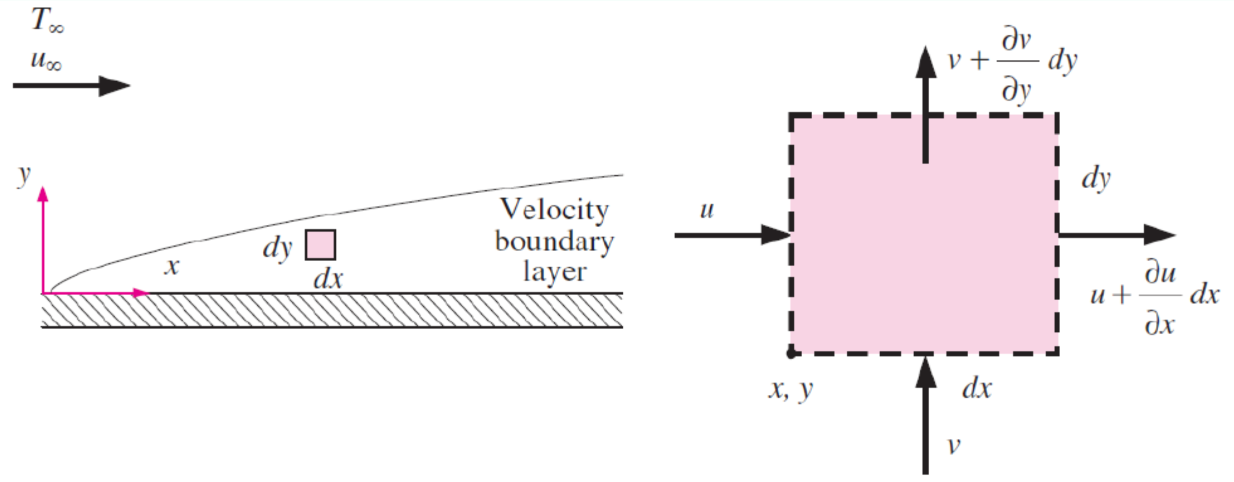
Prof. Ludwig Prandtl

- German Physicist, born in Bavaria (1875 - 1953)
- Father of aerodynamics
- Prof. of Applied Mechanics at Göttingen for 49 years (until his death)
- His work in fluid dynamics is still used today in many areas of aerodynamics and chemical engineering.

His discovery in 1904 of the Boundary Layer which adjoins the surface of a body moving in a fluid led to an understanding of skin friction drag and of the way in which streamlining reduces the drag of airplane wings and other moving bodies.

Assuming the flow/fluid to be:

- 2D, Steady
- Newtonian
- constant properties ($\rho, \mu, k,$ etc.)



Rate of mass flow into CV = Rate of mass flow out of CV

rate of fluid entering CV_{left}:

$$\rho u(dy \cdot 1)$$

rate of fluid leaving CV_{right}:

$$\rho(u + \frac{\partial u}{\partial x} dx)(dy \cdot 1)$$

$$\rho u(dy \cdot 1) + \rho v(dx \cdot 1) = \rho(u + \frac{\partial u}{\partial x} dx)(dy \cdot 1) + \rho(v + \frac{\partial v}{\partial y} dy)(dx \cdot 1)$$

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0}$$

Expressing Newton's second law of motion for the control volume:

$$(\text{Mass})\,(\text{Acceleration in } x) = (\text{Net body and surface forces in } x)$$

$$\delta m \cdot a_x = F_{\text{surface},x} + F_{\text{body},x} \tag{3}$$

$$\delta m = \rho(dx \cdot dy \cdot 1)$$

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \tag{4}$$

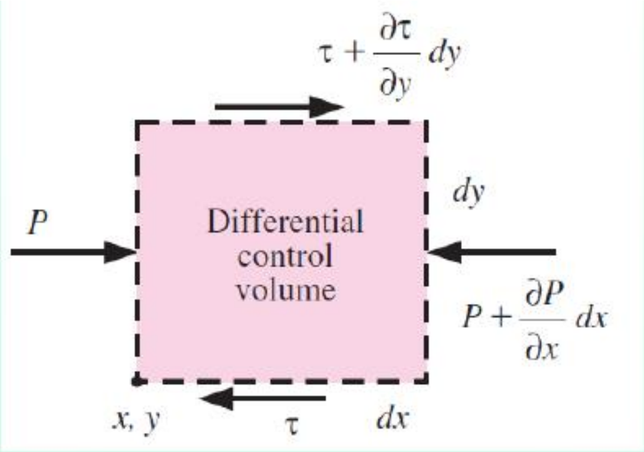
Steady state doesn't mean that acceleration is zero.
Ex: Garden hose nozzle

Body forces: gravity, electric and magnetic forces - $\propto$ volume.
Surface forces: pressure forces due to hydrostatic pressure and shear stresses due to viscous effects - $\propto$ surface area.

Viscous forces has two components:

1. normal to the surface- normal stress
   related to velocity gradients $\partial u/\partial x$ and $\partial v/\partial y$
2. along the wall surface - shear stress
   related to $\partial u/\partial y$

For simplicity, the normal stresses are neglected.



$$F_{\text{surface},x} = \left(\frac{\partial \tau}{\partial y} dy\right)(dx \cdot 1) - \left(\frac{\partial P}{\partial x} dx\right)(dy \cdot 1)$$

$$= \left(\frac{\partial \tau}{\partial y} - \frac{\partial P}{\partial x}\right)(dx \cdot dy \cdot 1)$$

$$\tau = \mu \left(\frac{\partial u}{\partial y}\right)$$

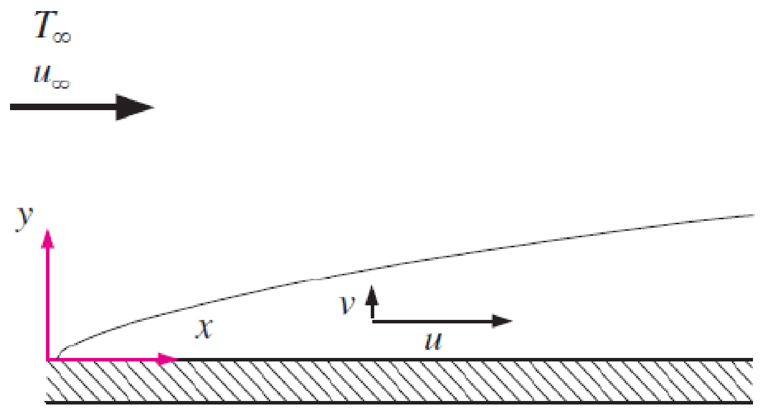
$$= \left(\mu \frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x}\right)(dx \cdot dy \cdot 1) \quad (5)$$

Combining Eqs. (3), (4) and (5):

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}\right) = \mu \frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x}$$

x-momentum equatio

$F = m \cdot a_x = F_{viscous} - F_{pressure}$



1) Velocity components:

$v \ll u$

2) Velocity grandients:

$\frac{\partial v}{\partial x} \approx 0, \frac{\partial v}{\partial y} \approx 0$

$\frac{\partial u}{\partial x} \ll \frac{\partial u}{\partial y}$

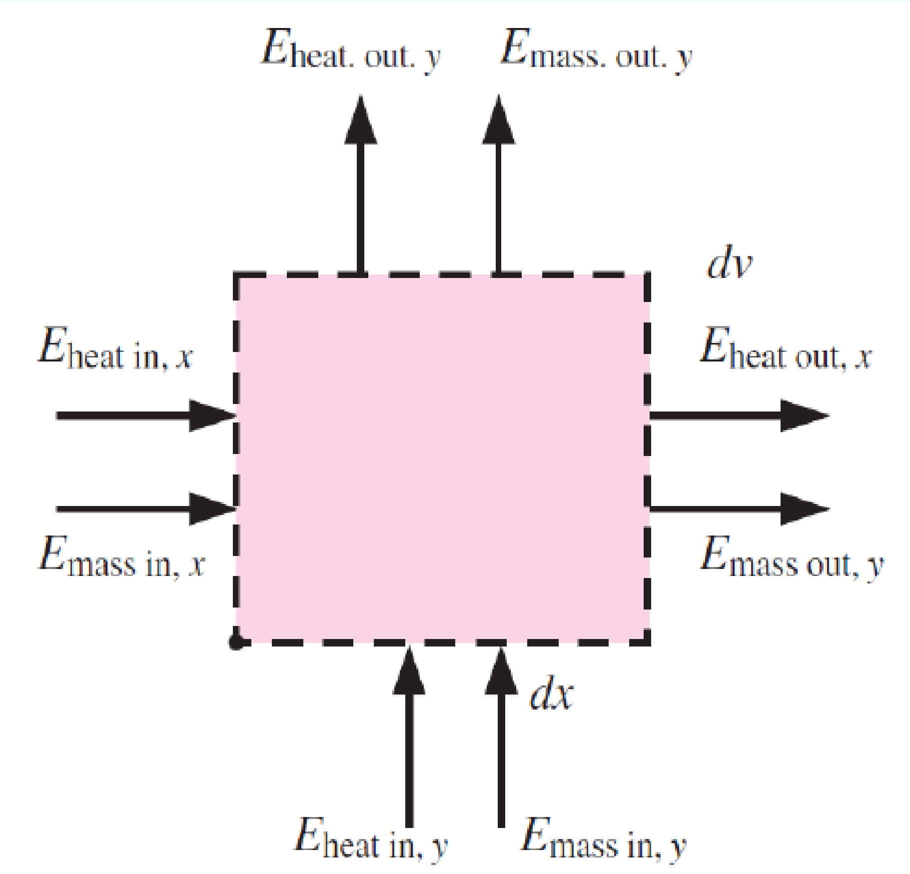
3) Temperature gradients:

$\frac{\partial T}{\partial x} \ll \frac{\partial T}{\partial y}$

$$\boxed{\frac{\partial P}{\partial y} = 0}$$

y-momentum equation

$$P = P(x) \implies \frac{\partial P}{\partial x} = \frac{dP}{dx}$$



$$(\dot{E}_{in} - \dot{E}_{out})_{\text{by mass}} + (\dot{E}_{in} - \dot{E}_{out})_{\text{by heat}} + (\dot{E}_{in} - \dot{E}_{out})_{\text{by work}} = 0 \tag{6}$$

Flowing fluid stream: is associated with enthalpy (internal energy and flow energy), potential energy (PE) and kinetic energy (KE)

$$
\begin{aligned}
(\dot{E}_{in} - \dot{E}_{out})_{\text{by heat},x} &= q_x - \left(q_x + \frac{\partial q_x}{\partial x} dx \right) \\
&= -\frac{\partial}{\partial x} \left(-k(dy \cdot 1) \frac{\partial T}{\partial x} \right) dx \\
&= k \frac{\partial^2 T}{\partial x^2} dx dy
\end{aligned}
$$

$$
\boxed{(\dot{E}_{in} - \dot{E}_{out})_{\text{by heat}} = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) dx dy} \quad (8)
$$

The total energy of a flowing stream is:

$$\dot{m}(h + pe + ke) = \dot{m}\left(h + gz + \frac{u^2 + v^2}{2}\right) = \dot{m}C_pT$$

KE: [m^2/s^2] ≈ J/kg. h is in kJ/kg. So, KE is expressed in kJ/kg by dividing it by 1000. KE term at low velocities is negligible. By similar argument, **PE** term is negligible.

$$(\dot{E}_{in} - \dot{E}_{out})_{\text{by mass},x} = (\dot{m}C_pT)_x - \left[(\dot{m}C_pT)_x + \frac{\partial(\dot{m}C_pT)_x}{\partial x}dx\right]$$

$$= -\frac{\partial[\rho u(dy \cdot 1)C_pT]}{\partial x}dx$$

$$(\dot{E}_{in} - \dot{E}_{out})_{\text{by mass}} = -\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right)dxdy \quad (7)$$

Combining Eqs. (6), (7), and (8):

$$\underbrace{\rho C_p \left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} \right)}_{\text{convection}} = \underbrace{k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)}_{\text{conduction}}$$

Net energy convected by the fluid out of CV

=

Net energy transfered into CV by conduction

[redacted]

When viscous shear stresses are not neglected, then:

$$\underbrace{\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)}_{\text{convection}}=\underbrace{k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)}_{\text{conduction}}+\underbrace{\mu\Phi}_{\text{viscous dissipation}}$$

where the viscous dissipation term is given as:

$$\mu\Phi=\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2+2\mu\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right]$$

This accounts for the rate at which mechanical work is irreversibly converted to thermal energy due to viscous effects in the fluid.

$$\boxed{\underbrace{u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}}_{\text{advection}}=\underbrace{\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)}_{\text{diffusion}}}$$

When the fluid is stationary, $u=v=0$:

$$k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)=0$$

Differential Convection Equations

Steady incompressible, laminar flow of a fluid with constant properties:

Continuity: $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Momentum: $$\rho \left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \right) = \mu \frac{\partial^2 u}{\partial y^2} - \frac{dP}{dx}$$

Energy: $$\rho C_p \left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$

with the boundary conditions

At $x = 0:$ $\quad u(0, y) = u_\infty, \quad T(0, y) = T_\infty$

At $y = 0:$ $\quad u(x, 0) = 0, \quad v(x, 0) = 0, T(x, 0) = T_s$

At $y \to \infty:$ $\quad u(x, \infty) = u_\infty, \quad T(x, \infty) = T_\infty$

Forced Convection

- Heat transfer between a fluid and solid surface is known as Convection and when the fluid is made to flow by external means like pump, fan, slope etc, the convection is called Forced Convection.
- Since energy transfer in convection takes place by movement of fluid molecules by picking up or going out heat energy, parameters like nature of flow, fluid velocity, viscous forces, etc have significant effects on heat transfer process.
- Hence, knowledge of differential equations for fluid flow like Continuity Equation, Momentum Equation (Navier Stoke's Equations) is vital. Based on these equations, information on fluid flow through the pipe and over flat plate like velocity profile, pressure drop etc, have been worked out.

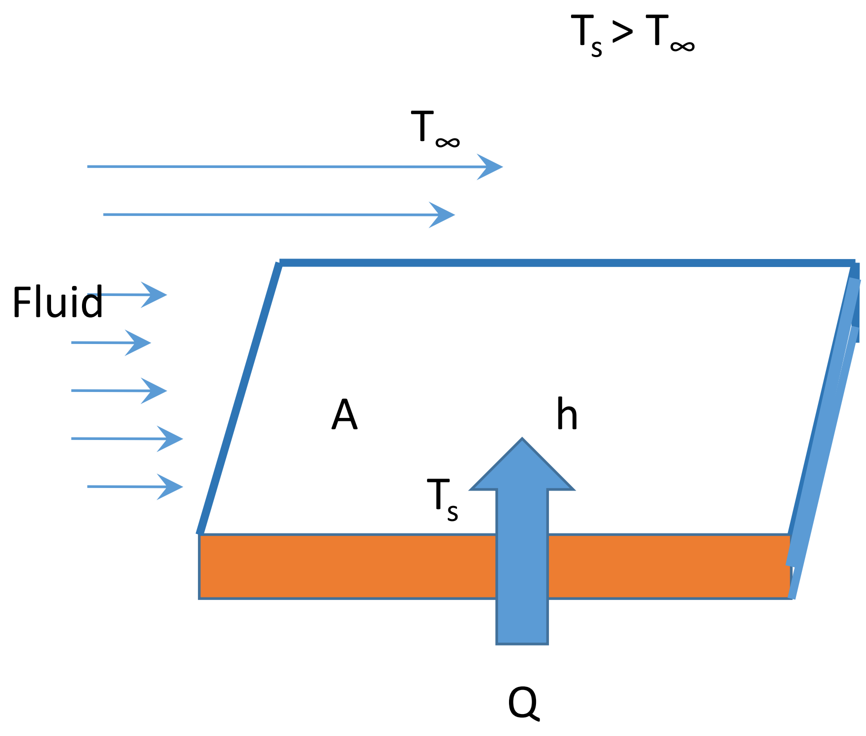
Convection

Heat Flow is found out from Newton's Law of Cooling as:

$$Q = hA(T_s - T_\infty)$$

Here h is neither property of surface nor that of fluid, but it is dependent on type of fluid flow, fluid properties, and vital dimension of surface or pipe.

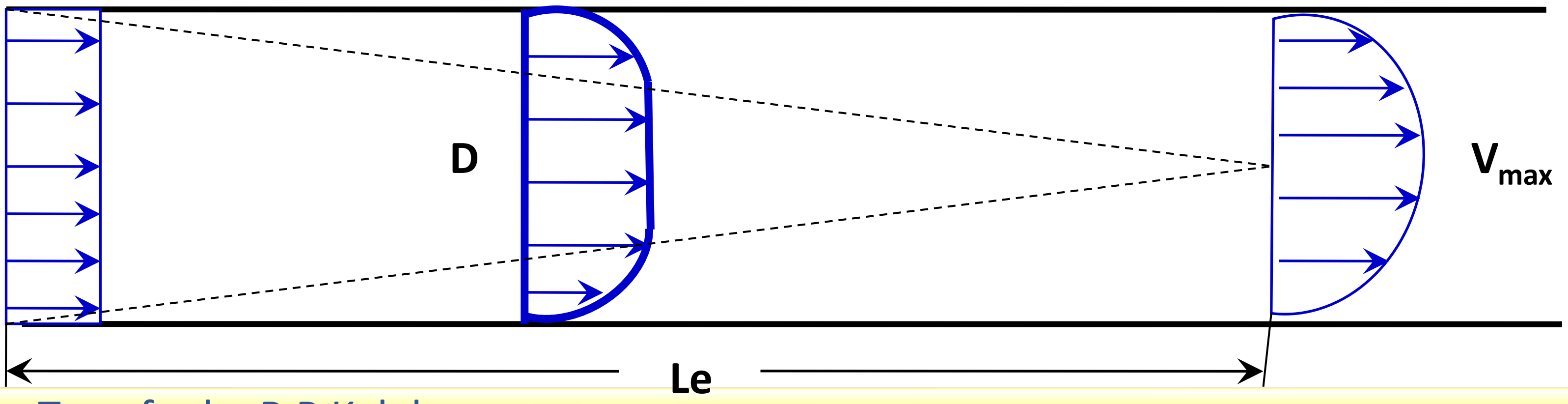
$$h = f(\rho, V, D/L, \mu, C_p, k)$$



Fluid Flow Through Pipe

- At the entrance of the tube, all fluid layers will have same velocity. When flow progresses, fluid layer in contact with surface tends to become stationary due to friction between tube surface and this layer of fluid.

- Due to viscous forces, this stationary layer retards the velocity of second layer towards the centre. Second layer retards the velocity of third layer and so on.



Fluid Flow Through Pipe

- Velocity of layers is proportional to the distance from the tube surface.
- For Law of Conservation of Mass to hold good, since velo is almost zero at the surface, it has to increase towards the centre as mass flow rate remains same at all sections of pipe.
- After certain distance from entrance, velocity profile develops fully and becomes steady.
- Profile becomes parabolic and does not change there after till the time flow remains Laminar.

Fully Developed Laminar Flow Through Pipe (Re <2000)

Entrance Length:
$$\frac{Le}{D} = 0.0575 Re$$

Local Velocity:
$$\frac{V}{V_{max}} = 1 - \left(\frac{r}{r_o}\right)^2 ;$$

$$where\ r_o\ is\ outer\ radius\ from\ centre$$

Average Velocity:
$$V_{av} = \frac{V_{max}}{2}$$

Friction Factor:
$$f = \frac{16}{Re} = \frac{\Delta P}{4\left(\frac{L}{D}\right)\left(\frac{\rho V^2}{2}\right)}$$

Turbulent Flow Through Pipe (Re >4000)

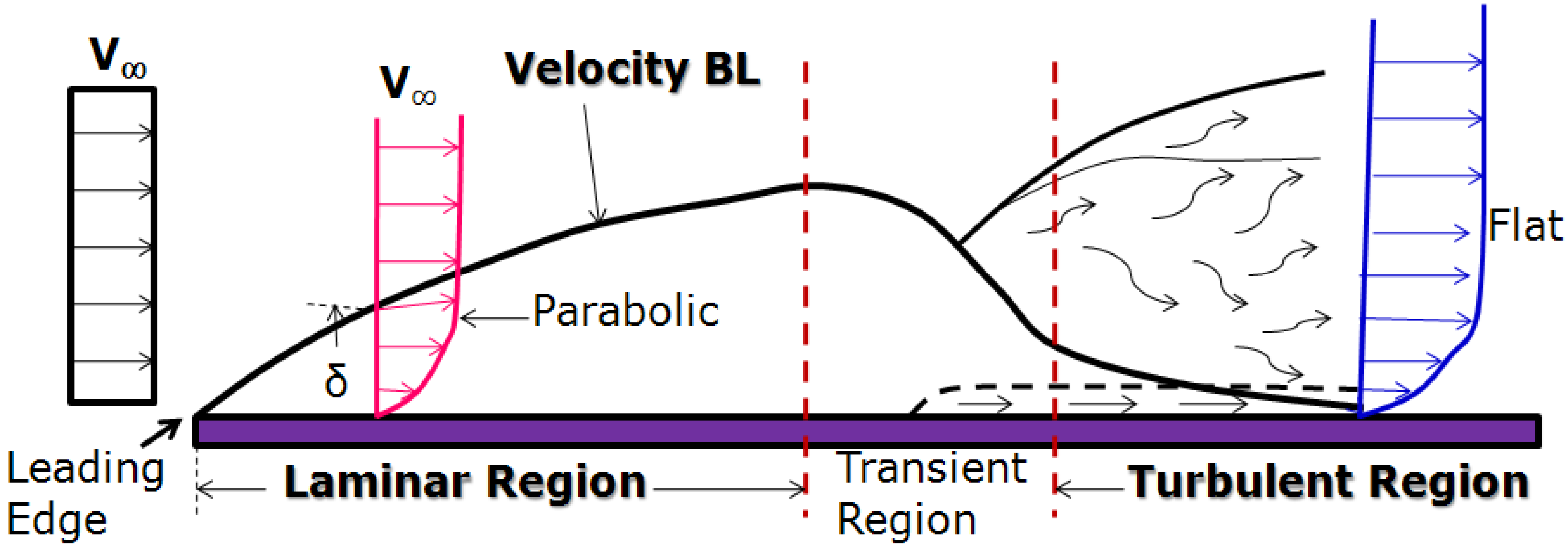
In turbulent flow, velocity profile quickly stabilizes due to large eddies formations. Hence entrance length is relatively smaller and velocity profile is flat in the core region of the pipe.

Friction Factor:

$$f = 0.046(\mathrm{Re})^{-0.2} = \frac{\Delta P}{4\left(\frac{L}{D}\right)\left(\frac{\rho V^2}{2}\right)}$$

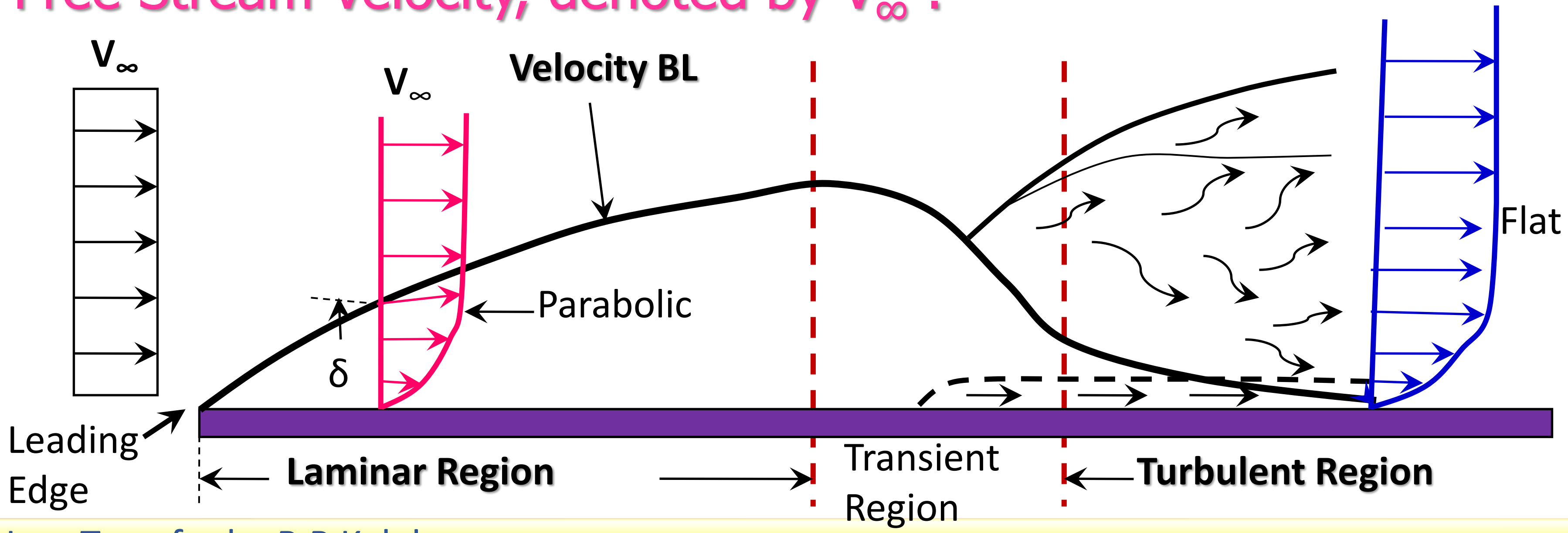
Flow Over Flat Plate : Velocity Boundary Layer

- When fluid flows over a flat plate, at the leading edge, all layers of fluid have same velocity.
- However, due to friction force, layer adjacent to plate comes to rest.



Flow Over Flat Plate : Velocity Boundary Layer

- Velocity of next layer is hence retarded by this stationary layer due to fluid viscous force.
- However, velocities of layers increase with distance from surface and beyond certain distance, it attains certain max steady value called Free Stream Velocity, denoted by V_{∞} .



Flow Over Flat Plate : Velocity Boundary Layer

The region normal to surface, in which velocity gradient exists, is known as Velocity BL / Hydrodynamic BL

- Thickness of Velo BL (δ) is defined as the distance normal to the surface, in which velo of layers varies from zero to 99% of the free stream velocity.

Fluid Flow in BL

- Laminar BL
- Transient BL
- Turbulent BL

Laminar Flow Over Flat Plate ($Re<3x10^5$)

Drag Coeff (C_f):

$$C_{f\,av} = \frac{1.328}{\sqrt{R_{eL}}}$$

Drag Force (F_D):

$$F_D = C_f \cdot \frac{\rho A V^2}{2}$$

Thickness of BL (δ) at x from leading edge

$$\delta_x = \frac{4.64\,x}{\sqrt{R_{ex}}}$$

Mass Flow Rate through BL at x

$$m_x = \frac{5}{8}\rho V \delta_x$$

Turbulent Flow Over Flat Plate (Re>5x10^5)

Drag Coeff (C_f):

$$C_{f\,av} = \frac{0.455}{\ln\left(R_{eL}^{2.58}\right)} - \frac{C_1}{R_{eL}};$$

$$where\, C_1 = 1050$$

Thickness of BL (δ) at x from leading edge

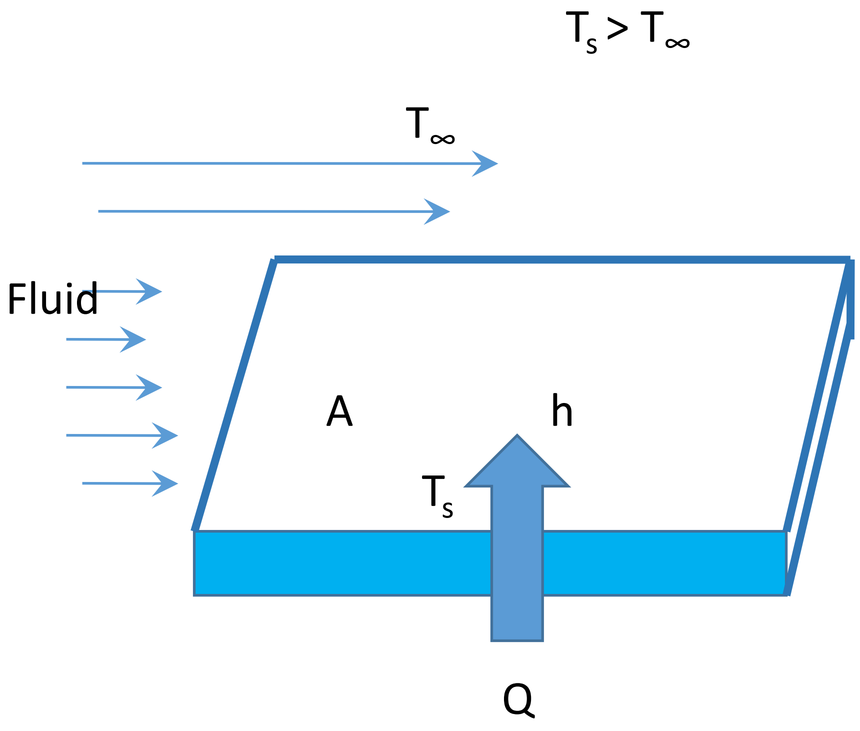
$$\delta_x = \frac{0.39.x}{\mathrm{Re}_x^{0.2}}$$

Convection

Heat Flow is determined as: $Q = hA(T_s - T_\infty)$

Here h is neither property of surface nor that of fluid But it is dependent on type of fluid flow, fluid properties, and vital dimension of surface or pipe.

$$h = f(\rho, V, D/L, \mu, C_p, k)$$



Convection

In practice, it is very difficult to estimate correct value of h and it becomes more complicated due to the fact that properties of all fluids vary with temp.

As the fluid flows over the surface, and if there is temp difference between fluid and surface, its tempin the BL changes. Accordingly, fluid properties change and hence h also changes. Thus on every loc on the surface along the fluid flow, we get different value of h. This value is known as Local Heat Transfercoefficient, denoted by h_x .

Hence, average h is found out as: $h_{av} = \frac{1}{L}\int_0^L h_x.dx$

Values of h (W/m^2K)

Free/Natural Convection with air:	5-15
Forced Convection with air :	10-500
Forced Convection with Water:	100-15000
Boiling of Water	1500-25000
Condensing Water Vapour	5000-100000

Forced Convection

- Since h = f (ρ, V, D/L, μ, C_p , k), it is very difficult to find relations of h because of large number of parameters involved.
- Such processes can be analyzed by Dimensional Analysis using Buckingham л Theorem.
- And we get the relations of the form :

$$\frac{hL}{k} = A\left(\frac{\rho VL}{\mu}\right)^a \left(\frac{\mu C_p}{k}\right)^b$$

Or $Nu = A\ (Re)^a\ (Pr)^b$

Dimensional Analysis

- If large No of variables take part in a process, it is very difficult or almost impossible to study the effects of variation of one or more variables on others.
- By dimensional analysis, these variables can be grouped in to manageable No of groups, say four or three or less so that effect of variation of each on others can be studied.

Buckingham л Theorem

- This theorem is used as a thumb rule for determining number of independent dimensionless groups that can be obtained from a set of variables taking part in a process.
- This Theorem states that the number of independent dimensionless groups that can be formed from a set of n variables having r basic/fundamental dimensions will be (n-r)

Dimensional Analysis For 'h'

- From different experiments, it has been seen that h in forced convection depends on ρ, V, L, μ, C_p and k.
- Hence, we can write h=f(ρ, V, L, μ, C_p , k) Or h=A(ρ^a, V^b, L^c, μ^d, $C_p^{\,e}$, k^f) where A, a, b, c, d, e, f are constants

Dimensional Analysis For ‘h’

Variables	Units	Dimensions
h	$W/m^2K = J/sm^2K = Nm/sm^2K = kg.m.m/s^2.s.m^2K = kg/s^3K$	$M.T^{-1}.t^{-3}$
ρ	Kg/m^3	$M.L^{-3}$
V	m/s	Lt^{-1}
L	m	L
μ	Kg/m.s	$M.L^{-1}t^{-1}$
C_p	$J/kg.K = m^2/s^2K$	$L^2.T^{-1}.t^{-2}$
k	$W/mK = kg.m/s^3K$	$M.L.T^{-1}.t^{-3}$

Dimensional Analysis For 'h'

→ $MT^{-1}t^{-3}=A[(ML^{-3})^a(Lt^{-1})^b(L)^c(ML^{-1}t^{-1})^d(L^2T^{-1}t^{-2})^e(MLT^{-1}t^{-3})^f]$

Equating powers of:

M: $1=a+d+f$ ……………………………………(1)

L: $0=-3a+b+c-d+2e+f$ …………………..(2)

T: $-1=-e-f$ or $1=e+f$ …………………(3)

t: $-3=-b-d-2e-3f$ ………………………………(4)

Dimensional Analysis For 'h'

- Let us obtain values of all constants in terms of only 2 constants, say 'a' & 'e'
- Hence, we obtain the eqn as under:

$h=A[\rho^a, V^a, L^{a-1}, \mu^{e-a}, C_p^{\,e}, k^{1-e}]$

$$\Rightarrow h = A\left[\left(\frac{\rho VL}{\mu}\right)^a \cdot \left(\frac{\mu Cp}{k}\right)^e \cdot \left(\frac{k}{L}\right)\right]$$

$$Or \quad \frac{hL}{k} = A\left(\frac{\rho VL}{\mu}\right)^a \cdot \left(\frac{\mu Cp}{k}\right)^e$$

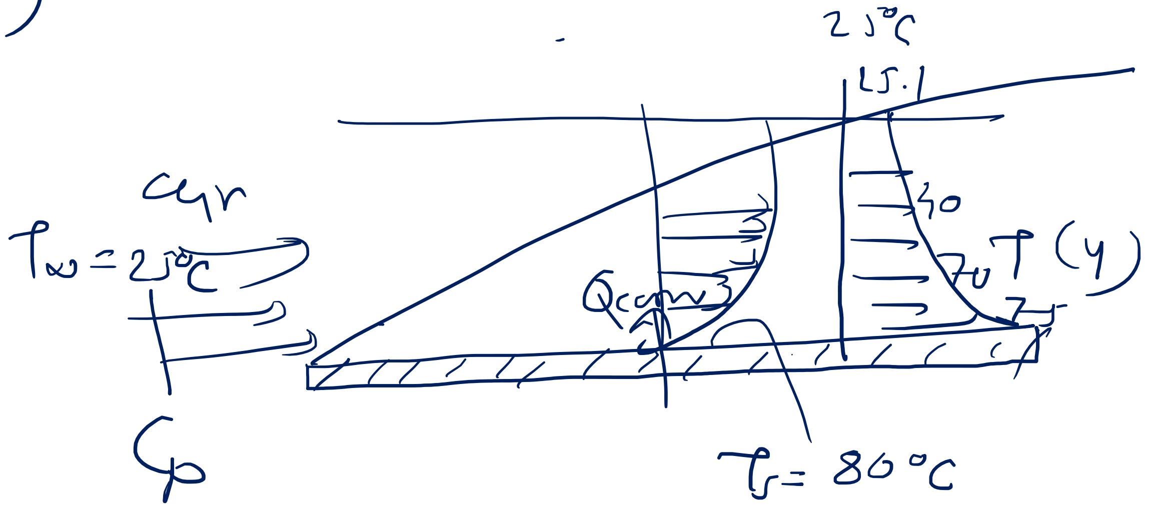
$$\Rightarrow Nu = A\,\mathrm{Re}^a \,.\, \mathrm{Pr}^e$$

finding property values (T_{mf})

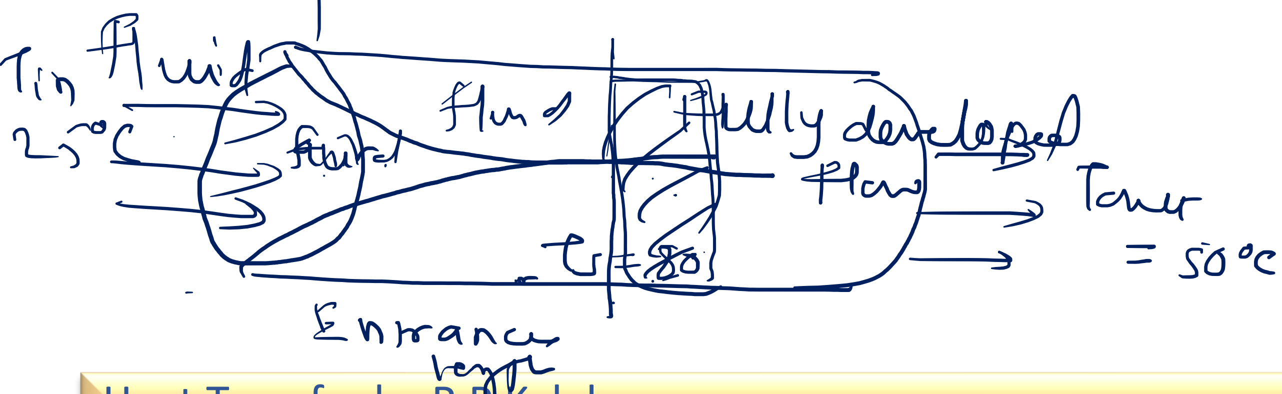
$$Pr = \frac{\mu C_p}{k} \qquad T_s \quad T_\infty$$

$$T_{avg} = \frac{T_s + T_\infty}{2} = T_{mf}$$

mean film



$$\frac{80+25}{2} = 52.5$$



T = ?

$$T_{avg} = \frac{25+50}{2} = T_{BM}$$

Bulk

Physical Significance of Dimensionless Parameters

Nusselt Number (Nu):

$$Nu = \frac{hL}{k} = \frac{hD}{k}$$

$$where\ L / D\ are\ characteristic\ length$$

$$= \frac{hL}{k} \cdot \frac{A\Delta T}{A\Delta T} = \frac{hA\Delta T}{\frac{kA\Delta T}{L}}$$

$$= \frac{Heat\ Transfer\ by\ Convection}{Heat\ Transfer\ by\ Conduction}$$

h can be found out from here

Prandtl Number:

$$Pr = \frac{\mu C_p}{k} = \frac{\frac{\mu}{\rho}}{\frac{k}{\rho C_p}} = \frac{v}{\alpha} = \frac{Kinematic\ Viscosity}{Thermal\ Diffusivity}$$

$$= \frac{Diffusion\ of\ Momentum\ through\ Fluid}{Diffusion\ of\ Heat\ through\ Fluid}$$

High Pr No means higher Nu and hence higher h; higher heat transfer

Pr No is the property of fluid as μ, C_p ,k are all properties of fluid and these are temp dependent

Prandtl Number:

For Liquid Metals: $Pr < 0.01$

For Air: $Pr \approx 1$

For Water: $Pr \approx 10$

For Heavy Oils: $Pr > 1$ lac

Reynold's No (Re):

$$Re = \frac{\rho VL}{\mu} = \frac{\rho VD}{\mu} = \frac{VL}{\nu} = \frac{VD}{\nu}\left(= \frac{4m}{\mu P}\right)$$

$$Re = \frac{\rho VL.V}{\mu.V} = \frac{\rho V^2}{\frac{\mu V}{L}}$$

$$= \frac{Inertia\,Force}{Viscous\,Force}$$

Peclet No (Pe):

$$Pe = \mathrm{Re}.\mathrm{Pr} = \frac{\rho VL}{\mu}.\frac{\mu C_p}{k} = \frac{\rho VC_p}{\frac{k}{L}}$$

$$= \frac{Mass Heat Flow Rate}{Heat Flow by Conduction per Unit Temp Diff}$$

When Pr is very small (of the order of 0.01), like for liquid metals, then as a practice, governing equation $Nu=A(Re)^a(Pr)^b$ is used as:

$$Nu=C(Pe)^n$$

This is only for convenience

Stanton No (St):

$$St = \frac{Nu}{Re.Pr} = \frac{hL}{\frac{k.\rho VL}{\mu} \cdot \frac{\mu C_p}{k}} = \frac{h}{\rho V C_p}$$

$$= \frac{Heat Flux in Convection per Unit Temp Diff}{Mass Heat Flow Rate}$$

In such cases, governing equation is used as: $St^n = C \text{ or } \left(\frac{Nu}{Re.Pr}\right)^n = C$

Reynold's Numbers

Flow through conduit/pipe

Laminar Flow : Re<2000

Turbulent Flow : Re>4000

Flow over flat plate/surface

Laminar Flow : Re $<3 \times 10^5$

Turbulent Flow : Re $>5 \times 10^5$

Correlations : Flow Through Pipe

For Laminar Flow (Re<2000)

Nu = 4.36 for const heat flux

Nu = 3.66 for const wall temp

Correlations : Flow Through Pipe

For Turbulent Flow (Re>4000)

$Nu = 0.023\ Re^{0.8}\ Pr^{0.4}$ for heating of fluid

$Nu = 0.023\ Re^{0.8}\ Pr^{0.3}$ for cooling of fluid

Above Equations are known as Dittus-Boelter Correlations

All properties of fluid are to be taken at Bulk Mean Temp

Hydraulic Diameter:

Characteristic Length for flow through pipe or conduit of different cross sections is taken as its hydraulic diameter (D_h), which is defined as:

$$D_h = \frac{4\,x\,Cross\,Sectional\,Area\,of\,Flow}{Wetted\,Perimeter} = \frac{4A}{P}$$

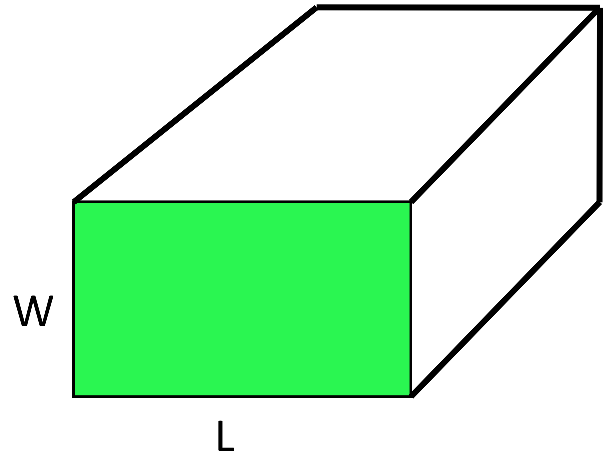
For circular tube of dia D:

$$D_h = \frac{4A}{P} = \frac{4.\frac{\pi}{4}D^2}{\pi D} = D$$

Hydraulic Diameter:

For rectangular cross section conduit

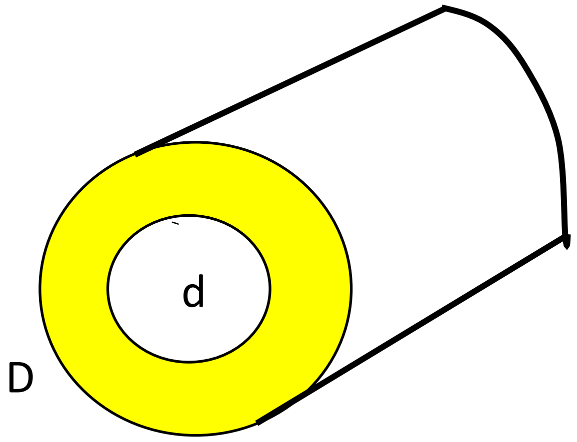
$$D_h = \frac{4A}{P} = \frac{4.LW}{2(L+W)}$$



For flow through annular space of outer dia D and inner dia d

$$D_h = \frac{4A}{P} = \frac{4.\left[\left(\frac{\pi}{4}D^2\right) - \left(\frac{\pi}{4}d^2\right)\right]}{\pi D + \pi d}$$

$$= \frac{\pi\left(D^2 - d^2\right)}{\pi(D+d)} = D - d$$



Flow of Liquid Metals Through Pipe (Low Pr)

$Nu = 5+0.025(Re.Pr)^{0.8}$ for const wall temp

$Nu = 4.82+0.0185(Pe)^{0.827}$ for const heat flux

Flow of Heavy Oil Through Pipe(High Pr)

$Nu=0.027Re^{0.8}Pr^{0.33}(\mu/\mu_w)^{0.14}$

(Sieder & Tate Relation)

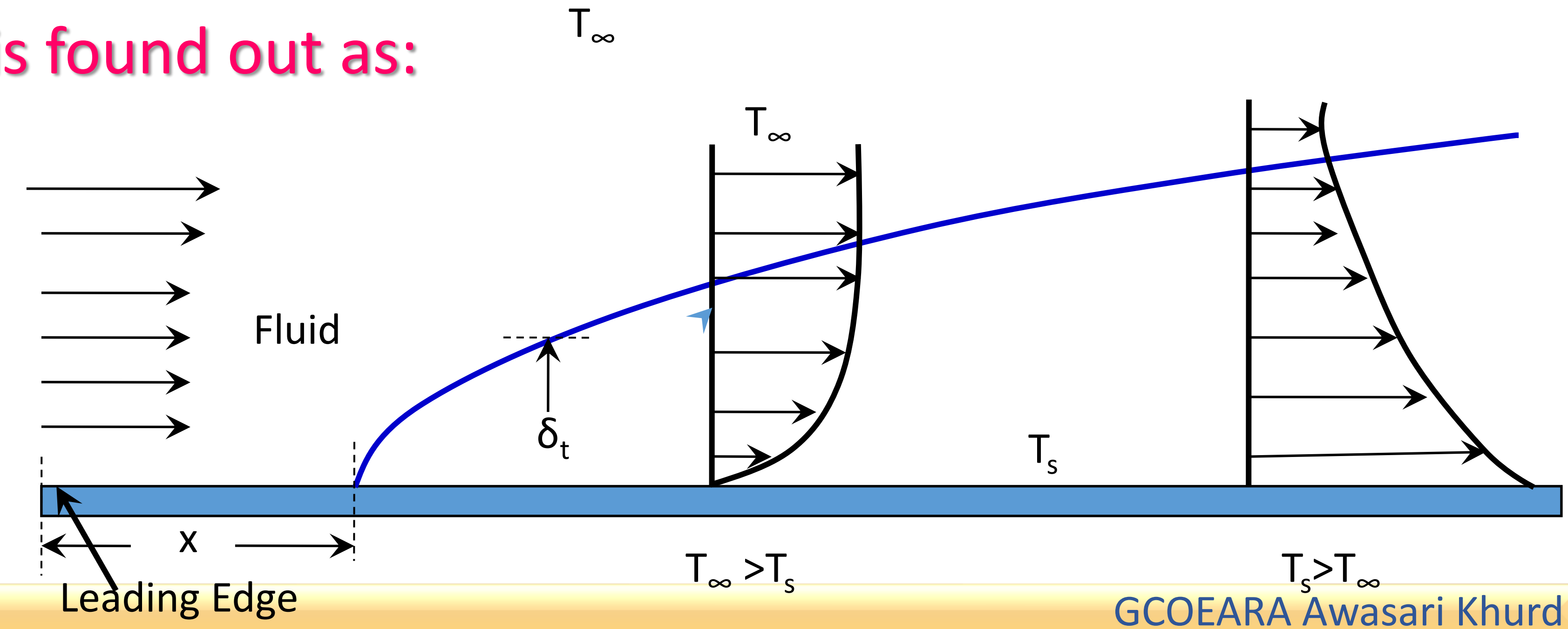
Flow Over Flat Plate

Thermal Boundary Layer

Thermal Boundary layer is the thin region over the surface, in which temp gradient exist.

Thickness of Thermal BL is found out as:

$$\frac{\delta_t}{\delta} = \frac{1}{1.026 Pr^{1/3}}$$



Laminar Flow Over Flat Plate

$$Local\ Nusselt\ No\ (at\ distance\ x\ from\ leading\ edge)$$

$$Nu_x = 0.332 Re_x^{1/2} . Pr^{1/3} \quad from\ dimensional\ analysis$$

$$To\ find\ Nu_{av} : We\ have$$

$$Nu_x = \frac{h_x . x}{K} = 0.332 Re_x^{1/2} . Pr^{1/3}$$

$$or\ h_x = 0.332 \frac{K}{x} \left(\frac{Vx}{v} \right)^{1/2} Pr^{1/3}$$

$$or\ h_x = 0.332 K . \left(\frac{V}{v} \right)^{1/2} Pr^{1/3} . x^{-1/2}$$

Laminar Flow Over Flat Plate

$$h_{av} = \frac{1}{L}\int_0^L h_x dx = \frac{1}{L}\int_0^L \left[0.332K\left(\frac{V}{\nu}\right)^{1/2} \mathrm{Pr}^{1/3} .x^{-1/2}\right] dx$$

$$= \frac{1}{L}\left[0.332K\left(\frac{V}{\nu}\right)^{1/2} \mathrm{Pr}^{1/3} \frac{x^{1/2}}{1/2}\right]_0^L$$

$$= 0.332\frac{K}{L}\left(\frac{VL}{\nu}\right)^{1/2} \mathrm{Pr}^{1/3} .2 = 2h_L$$

$$\frac{h_{av}.L}{K} = Nu_{av} = 0.664\mathrm{Re}^{\frac{1}{2}}.\mathrm{Pr}^{\frac{1}{3}}$$

Turbulent Flow Over Flat Plate

$Nu_x = 0.029\ Re_x^{0.8} .\ Pr^{0.334}$

$Nu = 0.0366\ Re^{0.8} .\ Pr^{0.334}$

Characteristic Length is the plate length (L) in the direction of fluid flow

All the fluid properties to be taken at mean film temp $T_{mean} = (T_s + T_\infty)/2$

Flow Across Horizontal Cylinder

$Nu_D = C\,(Re_D)^n$ for const heat flux

Hilpert's Relations

Re_D	C	n
40-4000	0.615	0.466
4000-40000	0.174	0.618

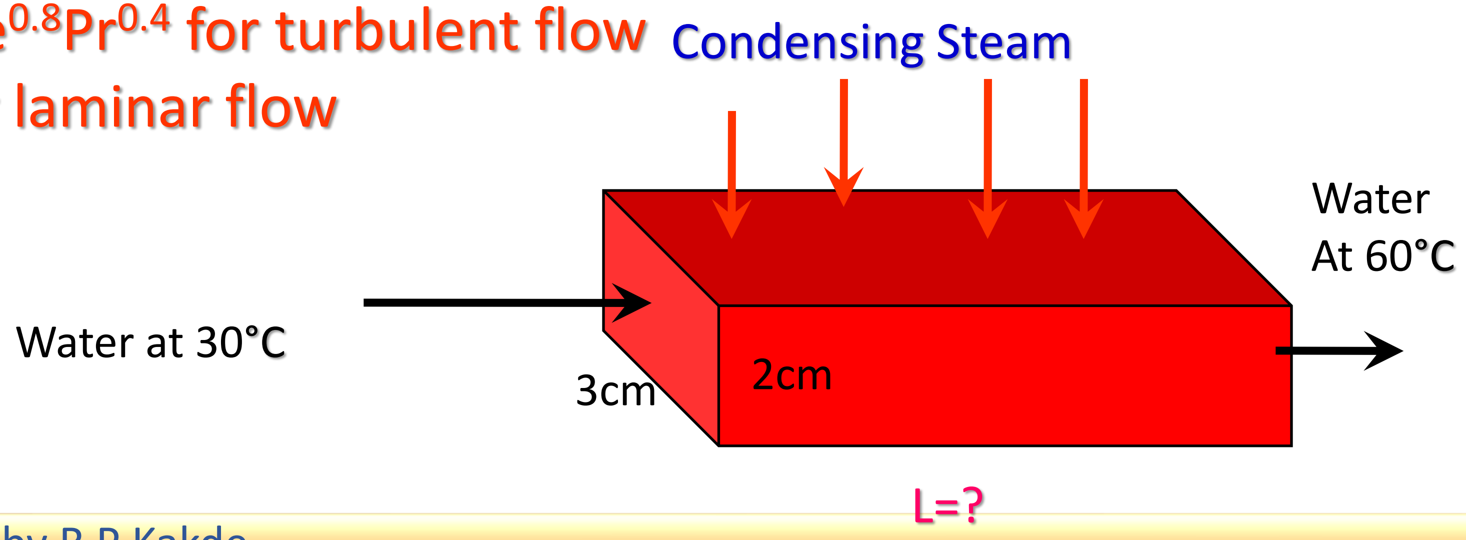
Q1: 65 kg/min of water is heated from 30°C to 60°C by passing it through a rectangular duct of 3cm x 2cm. The duct is heated by condensing the steam on its outer surface. Find the length of the duct required.

Properties of Water: ρ=995kg/m^3; μ=7.65x10^{-4}kg/ms; C_p=4.174kJ/kgK; k=0.623W/mK; ConductIllity of the Duct material=35W/mK

Use the following correlations:

Nu=0.023Re$^{0.8}$Pr$^{0.4}$ for turbulent flow

Nu=4.36 for laminar flow



We know that $Q = h\,A\,\Delta T = m\,C_p(T_e - T_i)$

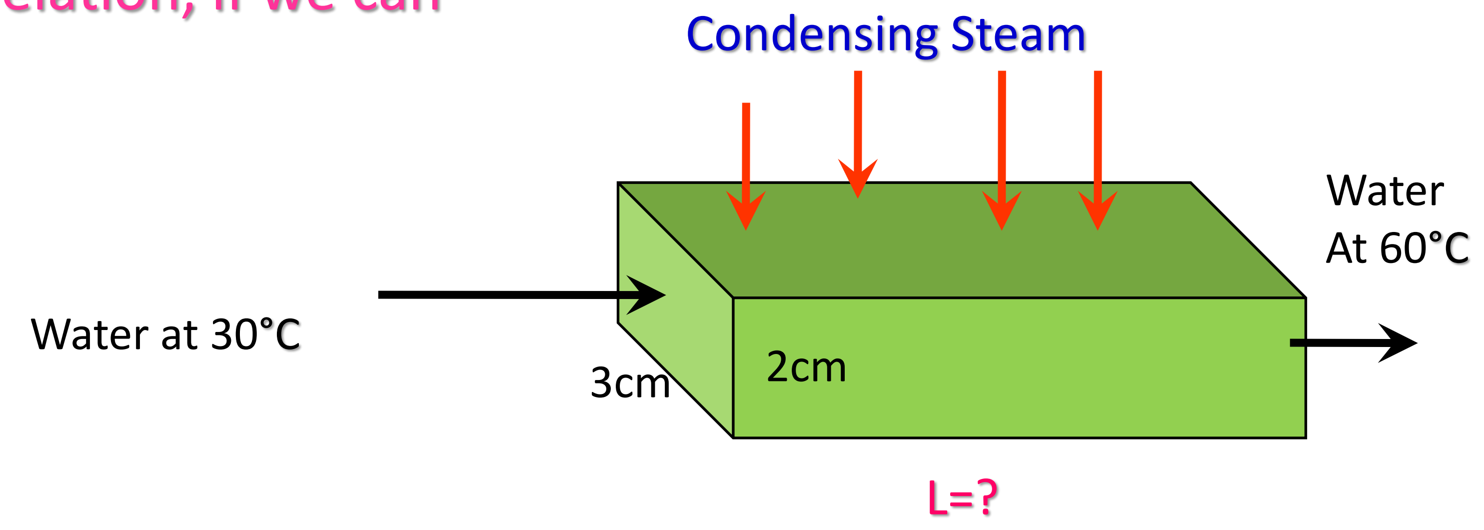
So Q=h (0.03+0.02)*2*L*[100-(30+60)/2]
=65/60[4174*(60-30)]

Hence L can be determined, provided h is known.

To determine h, we can use Nu relation, if we can Know which one to be used.

To find that, we should know whether flow is Laminar Or Turbulent

For that,
Re to be found out.



Solution (Contd):

$$\mathrm{Re} = \frac{\rho V D}{\mu};\ and\ we\ have\ to\ find\ V\ from\ m = \rho A V$$

$$and\ D\ from\ D_h = \frac{4A}{P}\ as\ conduit\ is\ NOT\ circular$$

$$D_h = \frac{4A}{P} = \frac{4*0.03*0.02}{(0.03+0.02)*2} = 0.024$$

$$m = \rho A V \Rightarrow V = \frac{65}{60*995*0.03*0.02} = 1.81 m/s$$

$$\mathrm{Re} = \frac{\rho V D_h}{\mu} = \frac{995*1.81*0.024}{7.65x10^{-4}} = 5.65x10^{4}$$

$$Since\ \mathrm{Re} = 5.65x10^{4} > 4000\ Flow\ is\ Turbulent$$

Solution (Contd):

$$Hence\ we\ have\ to\ use\ Nu = 0.023 Re^{0.8} Pr^{0.4}$$

$$Pr = \frac{\mu C_p}{k} = \frac{7.65x10^{-4} x4174}{0.623} = 5.125$$

$$Nu = \frac{hD_h}{k} = 0.023(5.65x10^4)^{0.8}(5.125)^{0.4}$$

$$\therefore\ h = \frac{0.623}{0.024} x0.023x63334.3x1.923 = 7271.48 W/m^2 K$$

$$7271.48(0.03+0.02)x2xL(100-45) = \frac{65}{60}x4174x(60-30)$$

$$\Rightarrow\ L = 3.38m\ Answer$$

Q2: Air at 20°C is flowing along a heated plate at 134°C with a velocity of 3m/s. The plate is 2m long. Heat transferred from first 40cm from the leading edge is 1.45kW. Determine the width of the plate.

Properties of air at 77°C: ρ=0.998kg/m^3; ν=20.76x10^{-6} m^2/s; C_p=1.009kJ/kgK; k=0.03W/mK.

Use the following correlation:

$N_{ux}=0.332\ Re^{0.5}\ Pr^{0.33}$

Solution:

(LINE OF APPROACH)

To determine width of the plate, we should find out area A transferring heat, since A=Width x Length (Length is gIIlen as 0.4m)

Area can be found out from Q=h A ΔT

Since Q & ΔT are known, we should find out h, which can be found out from gIIlen Nu_x relation.

Solution (Contd):

$$Re_{0.4} = \frac{VL}{v} = \frac{3x0.4}{20.76x10^{-6}} = 0.57803x10^{5}$$

$$Pr = \frac{\mu C_p}{k};\ Since \frac{\mu}{\rho} = v \ \Rightarrow \ \mu = \rho v$$

$$Hence\,Pr = \frac{\rho v C_p}{k}$$

$$= \frac{0.998x20.76x10^{-6}\,x1009}{0.03} = 0.697$$

Solution (Contd):

$$N_{uL} = \frac{h_L . L}{k} = 0.332(57803)^{0.5}(0.697)^{0.33}$$

$$h_L = \frac{0.03}{0.4} x0.332x240.4x0.887 = 5.313 W/m^2 K$$

We know that h_{av}=2h_L =2 x 5.313 =10.626

Hence Q=h A ΔT

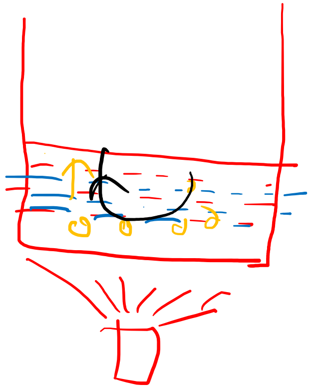
=10.626x0.4xWx(134-20) =1450 (glllen)

Therefore, width W=2.99m **Answer**

Forced convection – forcing fluid to enhance HT rate
[fan, Pump, comp. blower]

Free / Natural Convection

T↓ ρ↑ gravity Dominent



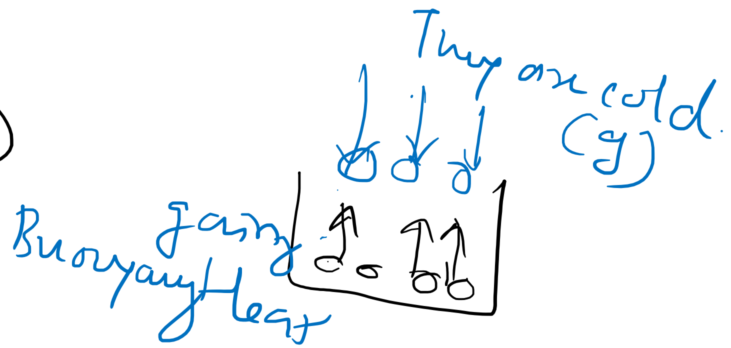
Down ward

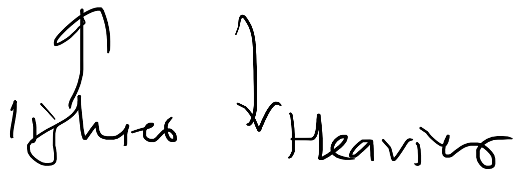
Fluid element
↓
Heat Gaining (T↑)
↓
Density (ρ) ↓
↳ Buoyancy predominent

Moving of fluid element in upward dirn
↑

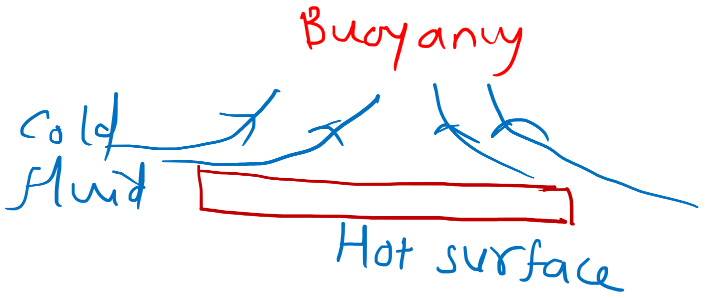
Natural Convection

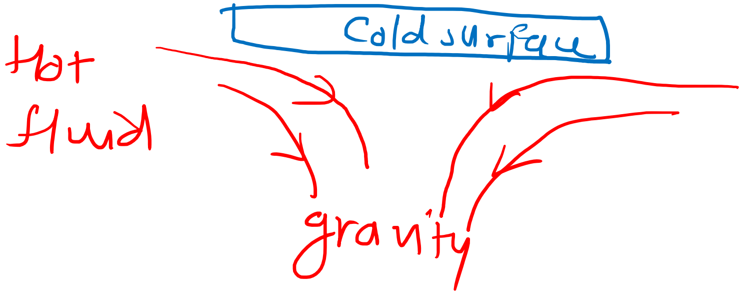
- When a fluid comes in contact with a hot surface, its molecules in the immediate vicinity receive heat from hot surface. (Q_s)
- Due to this, temp of molecules rise and then volume increases. (T↑)
- Therefore, fluid molecules become lighter and start rising. Buoyancy is dominant
- Their places are taken by heavier molecules, which also rise in similar way on taking energy from hot surface.



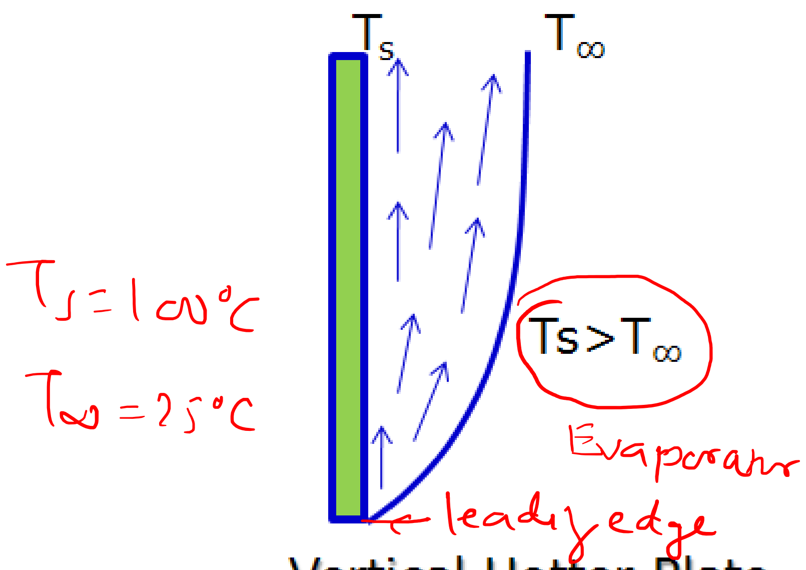


- Natural Convection
  - This way, natural motion in fluid molecules is set-in.
  - Transfer of heat from solid surface to fluid in this manner is called Free/Natural Convection.
  - When surrounding fluid is hotter than surface, heat transfer will be from fluid to surface.

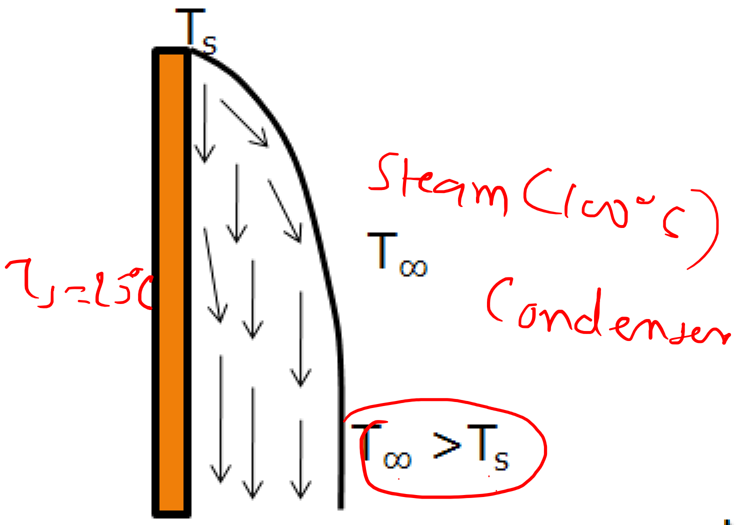




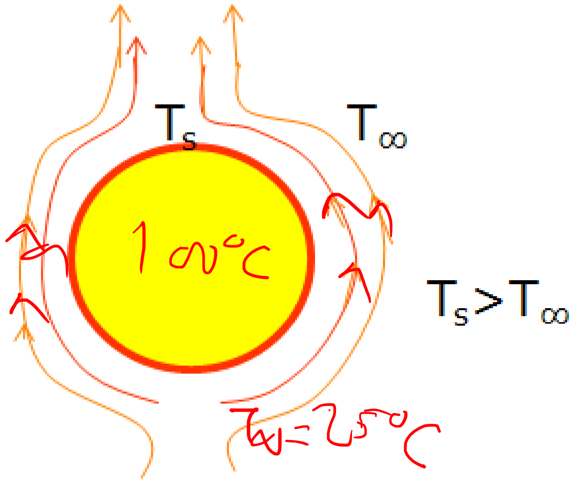
Natural Fluid Motion from Standard Surfaces



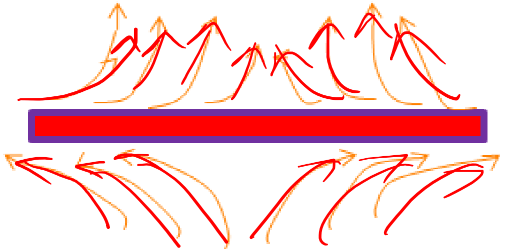
Vertical Hotter Plate



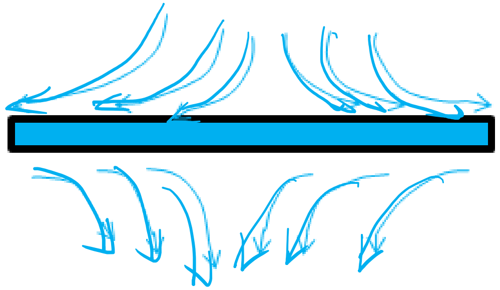
Vertical Colder Plate



Horizontal Hotter Cylinder



Horizontal Hotter Plate



Horizontal Colder Plate

Governing Equation In Natural Convection

In Natural Convection, $h = f(\rho, g, \beta, \Delta T, L, \mu, C_p, k)$

From dimensional analysis, we get the relation of following form:

$$\frac{hL}{k} = C\left[\left(\frac{\rho^2 . g . \beta . \Delta T . L^3}{\mu^2}\right)^a \left(\frac{\mu C_p}{k}\right)^b\right]$$

$$Nu = C(Gr)^a (Pr)^b \quad or$$

$$Nu = C(Gr.Pr)^n$$

Natural conv. - problem

This is the Governing Equation for Natural Convection

$Q_{conv} = h . A . (T_s - T_\infty)$

β = coeff. of thermal expansion

$= \frac{1}{\text{Temp (K) abs. scale}}$

°K

Physical Significance of Grashof No (Gr)

$$Gr = \frac{g.\beta.\Delta T.L^3.\rho^2}{\mu^2} = \frac{g.\beta.\Delta T.L^3}{\nu^2}$$

Rearranging terms, we get,

$$Gr = \frac{\left(\rho g \beta \Delta T L^3\right)\left(\rho V^2\right)}{\left(\mu V\right)^2}$$

$$Gr = \frac{Buoyancy\,Force \times Inertia\,Force}{\left(Viscous\,Force\right)^2}$$

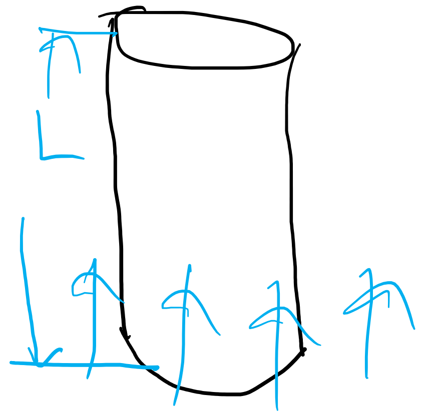
Grashoff No is the ratio of product of Buoyancy Force and Inertia Force to square of Viscous Force acting on fluid.

Correlations:Natural Convection

Vertical Plate & Cylinder

$Nu=0.56(Gr_L.Pr)^{1/4}$ for $10^4 < Gr.Pr < 10^8$

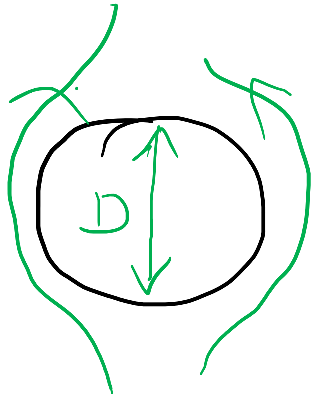
$=0.13(Gr_L.Pr)^{1/3}$ for $10^8 < Gr.Pr < 10^{12}$



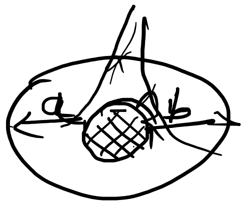
Horizontal Cylinder

$Nu=0.53(Gr_D.Pr)^{1/4}$ for $10^4 < Gr.Pr < 10^8$

$=0.13(Gr_D.Pr)^{1/3}$ for $10^8 < Gr.Pr < 10^{12}$



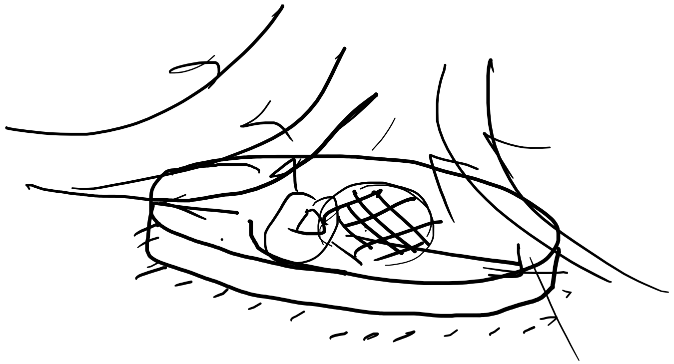
Correlations:Natural Convection



From Upper Surface of Square/Circular Plates

$Nu=0.54(Gr.\ Pr)^{1/4}$ for $10^5 < Gr.Pr < 2x10^7$

$=0.14(Gr\ .Pr)^{1/3}$ for $2x10^7 < Gr.Pr < 2x10^{10}$



From Lower Surface of Square/Circular Plates

$Nu=0.27(Gr.Pr)^{1/4}$ for $3x10^5 < Gr.Pr < 3x10^{16}$

Notes:-

1. Characteristic Length L=A/P
2. $\beta=1/T_{mean}$ in Kelvin
3. All properties of fluid to be taken at $T_{mean} =(T_{surface} +T_{fluid})/2$

Summary : Dimensionless Numbers

Conduction :

1. $B_i = \frac{hL}{k}$ 2. $F_o = \frac{a.t}{L^2}$

Biot No. Fourier No. (Transient conduction)

Bi, Nu

$\frac{hL}{k_{solid}}$ $\frac{hL}{k_{fluid}}$

Forced Convection:

3. $Nu = \frac{hL}{k}$ 4. $Re = \frac{\rho VL}{\mu}$ 5. $Pr = \frac{\mu C_p}{k}$ 6. $Pe = Re.Pr$ 7. $St = \frac{Nu}{Re.Pr}$

Forced convection

Natural Convection:

8. $Gr = \frac{g\beta\Delta TL^3}{v^2}$ 9. $Ra = Gr.Pr$ $Nu = \frac{hL}{k}$; $Pr = \frac{\mu C_p}{k}$

Natural convection

Mixed Convection: (0.3m/s≤ V ≤30m/s)

10. Graetz No $Gz = (Gr.Pr)\frac{d}{L}$

Q3: A circular disc insulated from other side of dia of 25cm is exposed to air at 20°C. If the disc (Open Surface) is maintained at 120°C, estimate heat transfer rate from it, when;

a) Disc is kept horizontal with (open) hot surface facing upwards
b) Disc is kept horizontal with (open) hot surface facing downwards
c) Disc is kept vertical

For air at 70°C, k=0.03; Pr=0.697; $v=2.076\times10^{-6}$

Use the following correlations:

$Nu=0.14(Gr.Pr)^{0.334}$ for upward/top surface

$Nu=0.27(Gr.Pr)^{0.25}$ for downward/bottom surface

$Nu=0.59(Gr.Pr)^{0.25}$ for vertical surface

Solution: Horizontal Plate-Convection from Top Surface

Heat Flow Rate Q=h.A.ΔT; h=? Nu=hL/k

$Nu=0.14(Gr.Pr)^{0.334}$

$$Gr = \frac{g\beta\Delta TL^3}{v^2}$$

$$\beta = \frac{1}{T_{mean}(K)} \Rightarrow \beta = \frac{1}{273+70} = 0.0029$$

$$L = \frac{A}{P} \Rightarrow L = \frac{\pi/4 \, D^2}{\pi D} = \frac{D}{4} = \frac{0.25}{4} = 0.0625$$

Solution: Horizontal Plate-Convection from Top Surface

$$Gr = \frac{9.81 x 1 x (120-20)(0.0625)^3}{(273+70).(2.076 x 10^{-6})^2} = 1.62 x 10^8$$

$$Nu = 0.14(1.62 x 10^8 x 0.697)^{1/3} = 68.51$$

$$= \frac{hL}{k} = \frac{h. x 0.0625}{0.03}$$

$$\Rightarrow h = 32.88 W/m^2 K$$

$$Q = hA\Delta T = 32.88 x \frac{\pi}{4}(0.25)^2(120-20) = 161 W$$

Solution: Horizontal Plate
Convection from Lower Surface

Heat Flow Rate Q=h.A.ΔT; h=? Nu=hL/k

$Nu=0.27(Gr.Pr)^{0.25}$

$$Gr = \frac{g\beta\Delta T L^3}{\nu^2}$$

$$\beta = \frac{1}{T_{mean}(K)} \qquad \Rightarrow \beta = \frac{1}{273+70} = 0.0029$$

$$L = \frac{A}{P} \qquad \Rightarrow L = \frac{\pi/4\, D^2}{\pi D} = \frac{D}{4} = \frac{0.25}{4} = 0.0625$$

Solution: Horizontal Plate-Convection from Lower Face

$$Gr = \frac{9.81 x 1 x (120 - 20)(0.0625)^3}{(273 + 70).(2.076 x 10^{-6})^2} = 1.62 x 10^8$$

$$Nu = 0.27(1.62 x 10^8 \; x 0.697)^{0.25} = 27.83$$

$$\Rightarrow \frac{hL}{k} = \frac{h.0.0625}{0.03} = 27.83$$

$$\Rightarrow h = 13.36 W/m^2 K$$

$$Q = hA\Delta T = 13.36 \frac{\pi}{4}(0.25)^2 (120 - 20) = 65.6 W$$

Solution: Vertical Plate

Heat Flow Rate Q=h.A.ΔT; h=? Nu=hL/k

$Nu=0.59(Gr.Pr)^{0.25}$

$$Gr = \frac{g\beta\Delta T L^3}{v^2}$$

$$\beta = \frac{1}{T_{mean}} \Rightarrow \beta = \frac{1}{273+70} = 0.0029$$

$$L = D = 0.25$$

Solution: Vertical Plate

$$Gr = \frac{9.81 x 1 x (120-20)(0.25)^3}{(273+70).(2.076 x 10^{-6})^2} = 103.6 x 10^8$$

$$Nu = 0.59(103.6 x 10^8 x 0.697)^{0.25} = 172$$

$$\Rightarrow \frac{hL}{k} = \frac{h.0.25}{0.03} = 172$$

$$\Rightarrow h = 20.64 W/m^2 K$$

$$Q = hA\Delta T = 20.64 \frac{\pi}{4}(0.25)^2 (120-20) = 101.3 W$$

Q4: A hot rectangular plate 5cm X 3cm maintained at 200°C is exposed to still air at 30°C. Calculate percentage increase in convectIIIe heat transfer rate if smaller side of the plate is held vertical than the bigger side. Neglect ITG of the thickness.

Use Correlation $Nu=0.59(Gr.Pr)^{0.25}$

Air properties at 115°C: density=$0.91 kg/m^3$; $C_p=1.009 kJ/kgK$; $\mu=22.65\times10^{-6}$; k=0.0331

Solution: Bigger Side (L=5cm) Vertical

$$Gr = \frac{\rho^2 g\beta\Delta T L^3}{\mu^2} = \frac{0.91^2 x 9.81 x 1 x (200-30)(0.05)^3}{(115+273)(22.65 x 10^{-6})^2}$$

$$= 8.67 x 10^5$$

$$\Pr = \frac{\mu C_p}{k} = \frac{22.65 x 10^{-6} x 1009}{0.0331} = 0.69$$

$$Nu = 0.59 (Gr.\Pr)^{0.25}$$

$$= 0.59 (8.67 x 10^5 x 0.69)^{0.25} = 16.41$$

Solution: Bigger Side (L=5cm) Vertical

$$Nu = \frac{h_L . L}{k} = 16.41$$

$$\Rightarrow h_L = 16.41 x \frac{0.0331}{0.05} = 10.86 W/m^2 K$$

$$Q = hA\Delta T$$

$$= 10.86 x 0.05 x 0.03 x 2 (200 - 30) = 5.54 W$$

Solution: Smaller Side (L=3cm) Vertical

Since Characteristic length has changed,
Grashof No will change, hence

$$Gr = \frac{\rho^2 g\beta\Delta T L^3}{\mu^2} = \frac{0.91^2 x 9.81 x 1 x (200-30)(0.03)^3}{(115+273)(22.65 x 10^{-6})^2} = 1.87 x 10^5$$

$$Nu = 0.59(Gr.\Pr)^{0.25} = 0.59(1.87 x 10^5 x 0.69)^{0.25} = 11.18$$

$$Nu = \frac{h_s.L}{k} = 11.18$$

$$\Rightarrow h_s = 11.18 x \frac{0.0331}{0.03} = 12.33 W/m^2K$$

Solution: Smaller Side (L=3cm) Vertical

$$Q = h_s A \Delta T$$

$$= 12.33 x 0.05 x 0.03 x 2 (200 - 30)$$

$$= 6.288 W$$

Increase in Heat Transfer Rate

$$Q = \frac{6.288 - 5.54}{5.54} x 100 = 13.5\%$$

Q5: A solid cylinder of steel (density=8000 Kg/m^3, C_p=0.42kJ/kgK) of 12cm dia and 30cm length at 380°C is suspended vertically in a large room at temp20°C. If the emissIllity of cylinder surface is 0.8, find total heat loss rate by the cylinder and initial rate of cooling.

Take properties of air at 200°C as follows:

C_p=1026J/kgK;ρ=0.746kg/m^3;k=0.0393W/mK

ν=34.85x10^{-6} m^2/s

Use the following correlations:

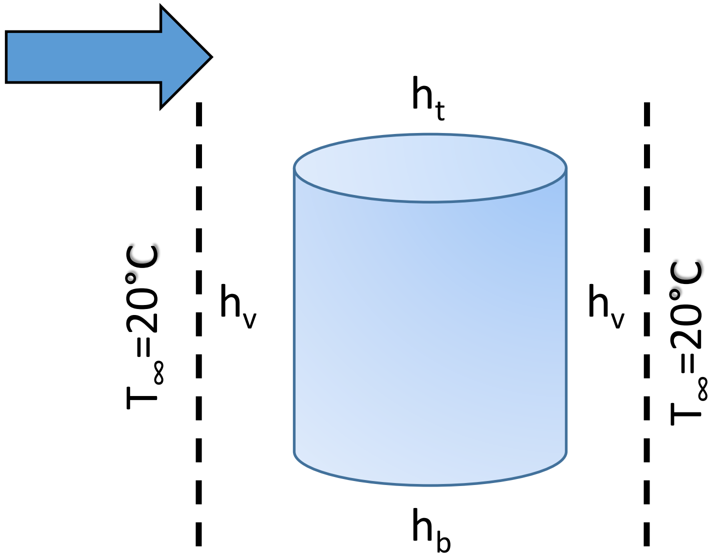
$Nu=0.56(Gr.Pr)^{0.25}$ for vertical surface

$Nu=0.27(Ra)^{0.25}$ for lower horizontal surface

$Nu=0.54(Ra)^{0.25}$ for upper horizontal surface

Solution:

Line of Approach



We have to find out heat flow rate Q=?

Heat flow will take place by convection and radiation.

Radiant heat flow $Q_r=\varepsilon_1\sigma A_1(T_s^4 - T_\infty^4)$

Heat flow by convection $Q_c=h\ A\ \Delta T$

Since h will be different for different surfaces i,e. h_t for top, h_b for bottom and h_v for vertical surfaces, we should first find out h_t, h_b and h_v by gIIlen Nu co-relations.

We can now find out Q_c for different surfaces. Add up all Q_c and Q_r to get total heat flow rate Q

Solution: **For Vertical Surface**

Mean Film Temp=(380+20)/2=200°C =473K

$$Gr = \frac{g\beta\Delta T L^3}{v^2}$$

$$= \frac{9.81 x (380-20) x 0.3^3}{473 x (34.85 x 10^{-6})^2} = 1.66 x 10^8$$

$$\Pr = \frac{\mu C_p}{k} = \frac{\rho v C_p}{k}$$

$$= \frac{0.746 x 34.85 x 10^{-6} x 1026}{0.0393} = 0.679$$

Solution: **For Vertical Surface**

$$Nu = 0.56(Gr\,Pr)^{0.25}$$

$$Nu = \frac{h_V L}{k}$$

$$= 0.56(1.66x10^8\,x0.679)^{0.25} = 57.69$$

$$h_V = \frac{0.0393x57.69}{0.3} = 7.56\,W/m^2K$$

Solution: **For Top Horizontal Surface**

$$CharacLengthL = \frac{A}{P} = \frac{\pi D^2}{4\pi D} = \frac{D}{4} = \frac{12}{4} = 3cm = 0.03m$$

$$Gr = \frac{g\beta\Delta TL^3}{v^2}$$

$$= \frac{9.81x(380-20)x0.03^3}{473x(34.85x10^{-6})^2} = 1.66x10^5$$

$$\Pr = \frac{\mu C_p}{k} = \frac{\rho v C_p}{k}$$

$$= \frac{0.746x34.85x10^{-6}x1026}{0.0393} = 0.679$$

Solution: **For Top Horizontal Surface**

$$Nu = 0.54(Gr\,\mathrm{Pr})^{0.25}$$

$$Nu = \frac{h_t L}{k} = 0.54(1.66x10^5\;x0.679)^{0.25} = 9.89$$

$$\Rightarrow\; h_t = \frac{0.0393x9.89}{0.03} = 12.96\;W/m^2K$$

Solution: **For Bottom Horizontal Surface**

$$CharacLengthL = \frac{A}{P} = \frac{\pi D^2}{4\pi D} = \frac{D}{4} = \frac{12}{4} = 3cm = 0.03m$$

$$Gr = \frac{g\beta\Delta TL^3}{v^2}$$

$$= \frac{9.81x(380-20)x0.03^3}{473x(34.85x10^{-6})^2} = 1.66x10^5$$

$$\Pr = \frac{\mu C_p}{k} = \frac{\rho v C_p}{k}$$

$$= \frac{0.746x34.85x10^{-6}x1026}{0.0393} = 0.679$$

Solution:

For Bottom Horizontal Surface

$$Nu = 0.27(Gr\,\mathrm{Pr})^{0.25}$$

$$Nu = \frac{h_b L}{k} = 0.27(1.66x10^5\,x0.679)^{0.25}$$

$$h_b = 6.48W/m^2K$$

Hence total heat flow by convection

$$Qc = h_v.\pi DL(T_s - T_\infty) + h_t.\frac{\pi}{4}D^2.(T_s - T_\infty) + h_b\frac{\pi}{4}D^2.(T_s - T_\infty)$$

$$= 7.56x\pi x0.12x0.3(380-20) + 12.96x\frac{\pi}{4}.0.12^2(380-20)$$

$$+ 6.48x\frac{\pi}{4}.0.12^2(380-20) = 386.76W$$

Heat loss by Radiation $Q_r = \varepsilon_1 \sigma A_1(T_s^4 - T_\infty^4)$

$$Q_r = 0.8x5.67x10^{-8}x(\pi DL + 2x\frac{\pi}{4}D^2)(653^4 - 293^4)$$

$$= 1073.35W$$

Hence total heat flow by convection and Radiation →

$$Q = Qc + Qr = 386.76 + 1073.35 = 1460 W$$

$$To\ obtain\ Initial\ Rate\ of\ Cooling\ Q = -mC_p \frac{dT}{dt}$$

$$m = \rho V = \frac{8000 x \pi (0.12)^2 x 0.3}{4} = 27.13 kg$$

$$\therefore \frac{dT}{dt} = \frac{1460}{420 x 27.13} = 0.128^\circ C/\sec = 7.69^\circ C/\min$$